# Correction of Spatially Varying Image and Video Motion Blur using a Hybrid Camera

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#### Abstract

We describe a novel approach to reduce spatially-varying motion blur in video and images using a hybrid camera system. A hybrid camera is a standard video camera that is coupled with an auxiliary low-resolution camera sharing the same optical path but capturing at a significantly higher frame rate. The auxiliary video is temporally sharper but at a lower resolution, while the lower-frame-rate video has higher spatial resolution but is susceptible to motion blur.

Our deblurring approach uses the data from these two video streams to reduce spatially-varying motion blur in the high-resolution camera with a technique that combines both deconvolution and super-resolution. Our algorithm also incorporates a refinement of the spatially-varying blur kernels to further improve results. Our approach can reduce motion blur from the high-resolution video as well as estimate new high-resolution frames at a higher framerate. Experimental results on a variety of inputs demonstrate notable improvement over current state-of-the-art methods in image/video deblurring.

## I. INTRODUCTION

This paper introduces a novel approach to reduce spatially-varying motion blur in video footage. Our approach uses a hybrid camera framework first proposed by Ben-Ezra and Nayar [6], [7]. A hybrid camera system simultaneously captures a high-resolution video together with a low-resolution video that has denser temporal sampling. The hybrid camera system is designed such that the two videos are synchronized and share the same optical path. Using the information in these two videos, our method has two aims: 1) to deblur the frames in the high-resolution video, and 2) to estimate new high-resolution video frames at a higher temporal sampling.

While high-resolution, high-frame-rate digital cameras are becoming increasingly more affordable (e.g.  $1960 \times 1280$  at 60fps are now available at consumer prices), the hybrid-camera design remains promising. Even at 60fps, high-speed photography/videography is susceptible to motion blur artifacts. In addition, as the frame-rate of high-resolution cameras increase, low-resolution camera frame-rate speeds increase accordingly with cameras available now with over 1000fps at lower-resolution. Thus, our approach has application to ever increasing temporal imaging. In addition, the use of hybrid cameras, and hybrid-camera like designs, have been demonstrated to offer other advantages over single view cameras including object segmentation and matting [7], [35], [36], depth estimation [31] and high dynamic range imaging [1]. The ability to perform object segmentation is key in deblurring moving objects as demonstrated by [7] and our own work in Section V.



Fig. 1. Tradeoff between resolution and frame rates. Top: Image from a high-resolution, low-frame-rate camera. Bottom: Images from a low-resolution, high-frame-rate camera.

The previous work in [6], [7] using a hybrid camera system focused on correcting motion blur in a single image targeting globally invariant motion blur. In this paper, we address the broader problem of correcting spatially-varying motion blur and aim to deblur temporal sequences. In addition, our work achieves improved deblurring performance by more comprehensively exploiting the available information acquired in the hybrid camera system, including optical flow, back-projection constraints between low-resolution and high-resolution images, and temporal coherence along image sequences. In addition, our approach can be used to increase the frame rate of the high-resolution camera by estimating the missing frames.

The central idea in our formulation is to combine the benefits of both deconvolution and super-resolution. Deconvolution of motion blurred, high-resolution images yields high-frequency details, but with ringing artifacts due to lack of low-frequency components. In contrast, super-resolution-based reconstruction from low-resolution images recovers artifact-free low-frequency results that lack high-frequency detail. We show that the deblurring information from deconvolution and super-resolution are complementary to each other and can be used together to improve deblurring performance. In video deblurring applications, our method furthermore capitalizes on additional deconvolution constraints that can be derived from consecutive video frames. We demonstrate that this approach produces excellent results in reducing spatially-varying motion blur. In addition, the availability of the low-resolution imagery and subsequently derived motion



Fig. 2. The processing pipeline of our system. Optical flows are first calculated from the Low-Resolution, High-Frame-Rate (LR-HFR) video. From the optical flows, spatially-varying motion blur kernels are estimated (Section III-B). Then the main algorithm performs an iterative optimization procedure which simultaneously deblurs the High-Resolution, Low-Frame-Rate (HR-LFR) image/video and refines the estimated kernels (Section IV). The output is a deblurred HR-LFR image/video. For the case of deblurring a moving object, the object is separated from the background prior to processing (Section V). In the deblurring of video, we can additionally enhance the frame rate of the deblurred video to produce a High-Resolution, High-Frame-Rate (HR-HFR) video result (Section VI).

vectors further allows us to perform estimate new temporal frames in the high-resolution video, which we also demonstrate.

A shorter version of this work appeared in [47]. This journal version extends our conference work with greater discussion of the deblurring algorithm, further technical details of our implementation, and additional experimentations. In addition, a method to estimate new temporal frames in the high-resolution video is presented in Section VI together with supporting experiments in Section VII.

The processing pipeline of our approach is shown in Figure 2, which also relates process components to their corresponding section in the paper. The remainder of the paper is organized as follows: Section II discusses related work; Section III describes the hybrid camera setup and the constraints on deblurring available in this system; Section IV describes our overall deconvolution formulation expressed in a maximum a posteriori (MAP) framework; Section V discusses how to extend our framework to handle moving objects; Section VI describes how to perform temporal upsampling with our framework; Section VII provides results and comparisons with other current work, followed by a discussion and summary in Section VIII.

## II. RELATED WORK

Motion deblurring can be cast as the deconvolution of an image that has been convolved with either a global motion point spread function (PSF) or a spatially-varying PSF. The problem is inherently ill-posed as there are a number of unblurred images that can produce the same blurred image after convolution. Nonetheless, this problem is well studied given its utility in photography and video capture. The following describes several related work.

**Traditional Deblurring** The majority of related work involves traditional blind deconvolution which simultaneously estimates a global motion PSF and the deblurred image. These methods include well-known algorithms such as Richardson-Lucy [40], [33] and Wiener deconvolution [50]. For a survey on blind deconvolution readers are referred to [20], [19]. These traditional approaches often produce less than desirable results that include artifacts such as ringing.

**PSF Estimation and Priors** A recent trend in motion deblurring is to either constrain the solution of the deblurred image or to use auxiliary information to aid in either the PSF estimation or the deconvolution itself (or both). Examples include work by Fergus *et al.* [17], which used natural image statistics to constrain the solution to the deconvolved image. Raskar *et al.* [38] altered the shuttering sequence of a traditional camera to be make the PSF more suitable for deconvolution. Jia [23] extracted an alpha mask of the blurred region to aid in PSF estimation. Dey *et al.* [15] modified the Richardson-Lucy algorithm by incorporating total variation regularization to suppress ringing artifacts. Levin *et al.* [28] introduced gradient sparsity constraints to reduce ringing artifacts. Yuan *et al.* [53] proposed a multiscale non-blind deconvolution approach to progressively recover motion blurred details. Shan *et al.* [41] studied the relationship between estimation errors and ringing artifacts, and proposed the use of a spatial distribution model of image noise together with a local prior that suppresses ringing to jointly improve global motion deblurring.

Other recent approaches use more than one image to aid in the deconvolution process. Bascle *et al.* [5] processed a blurry image sequence to generate a single unblurred image. Yuan *et al.* [52] used a pair of images, one noisy and one blurred. Rav-Acha and Peleg [39] consider images that have been blurred in orthogonal directions to help estimate the PSF and constrain the resulting image. Chen and Tang [11] extend the work of [39] to remove the assumption of orthogonal blur directions. Bhat *et al.* [8] proposed a method that uses high-resolution photographs to enhance

low-quality video, but this approach is limited to static scenes. Most closely related to ours is the work of Ben-Ezra and Nayar [6], [7], which used an additional imaging sensor to capture high-frame-rate imagery for the purpose of computing optical flow and estimating a global PSF. Li *et al.* [31] extend the work of Ben-Ezra and Nayar [6], [7] by using parallel cameras with different frame rates and resolutions, but their work targets depth map estimation and not deblurring.

The aforementioned approaches assume the blur to arise from a global PSF. Recent work addressing spatially-varying motion blur include that of Levin [27], which used image statistics to correct a single motion blur on a stable background. Bardsley *et al.* [4] segmented an image into regions exhibiting similar blur, while Cho *et al.* [12] used two blurred images to simultaneously estimate local PSFs as well as deconvolve the two images. Ben-Ezra and Nayar [7] demonstrated how the auxiliary camera could be used to separate a moving object from the scene and apply deconvolution to this extracted layer. These approaches [27], [4], [12], [7], however, assume the motion blur to be globally invariant within each separated layer. Work by Shan *et al.* [42] allows the PSF to be spatially-varying; however, the blur is constrained to that from rotational motion. Levin *et al.* [30] proposed a parabolic camera designed for deblurring images with 1D object motion. During exposure, the camera moves in a manner that allows the resulting image to be deblurred using a single deconvolution kernel.

**Super-resolution and upsampling** The problem of super-resolution can be considered as a special case of motion deblurring in which the blur kernel is a low-pass filter that is uniform in all motion directions. High-frequency details of a sharp image are therefore completely lost in the observed low-resolution image. There are two main approaches to super-resolution: image hallucination based on training data and image super-resolution computed from multiple low-resolution images. Our work is closely related to the latter approach, which is reviewed here. The most common technique for multiple image super-resolution is the back-projection algorithm proposed by Irani and Peleg [21], [22]. The back-projection algorithm is an iterative refinement procedure that minimizes the reconstruction errors of an estimated high-resolution image through a process of convolution, downsampling and upsampling. A brief review that includes other early work on multiple image super-resolution is given in [10]. More recently, Patti *et al.* [37] proposed a method to align low-resolution video frames with arbitrary sampling lattices to reconstruct a high-resolution video. Their approach also uses optical flow for alignment and PSF estimation. These estimates, however, are global and do not consider local object motion. This work was

extended by Elad and Feuer [16] to use adaptive filtering techniques. Zhao and Sawhney [55] studied the performance of multiple image super-resolution against the accuracy of optical flow alignment and concluded that the optical flows need to be reasonably accurate in order to avoid ghosting effects in super-resolution results. Shechtman et al. [43] proposed space-time superresolution in which multiple video cameras with different resolutions and frame rates are aligned using homographies to produce outputs of either higher temporal and/or spatial sampling. When only two cameras are used, this approach can be considered a demonstration of a hybrid camera, however, this work does not address the scenario where severe motion-blur is present in the highresolution, low-frame-rate camera. Sroubek et al. [45] proposed a regularization framework for solving the multiple image super-resolution problem. This approach also does not consider local motion blur effects. Recently, Agrawal [2] proposed a method to increase the resolution of images that have been deblurred using a coded exposure system. Their approach can also be considered as a combination of motion deblurring and super-resolution, but is limited to translational motion. While various previous works are related in part, our work is unique in its **Our** work focus on spatially-varying blur with no assumption on global or local motion paths. Moreover, our approach takes full advantage of the rich information available from the hybrid camera system, using techniques from both deblurring and super-resolution together in a single MAP framework. Specifically, our approach incorporate spatially-varying deconvolution together with back-projection against the low-resolution frames. This combined strategy produces deblurred images with less ringing than traditional deconvolution, but with more detail than approaches using regularization and prior constraints. As with other deconvolution methods, we cannot recovery frequencies that have been completely loss due to motion blur and downsampling. A more detail discussion on our approach is provided in Section IV-D.

## III. HYBRID CAMERA SYSTEM

The advantages of a hybrid camera system are derived from the additional data acquired by the low-resolution, high-frame-rate (LR-HFR) camera. While the spatial resolution of this camera is too low for many practical applications, the high-speed imagery is reasonably blur free and is thus suitable for optical flow computation. Figure 1 illustrates an example. Since the cameras are assumed to be synchronized temporally and observing the same scene, the optical flow corresponds to the motion of the scene observed by the high-resolution, low-frame-rate



Fig. 3. Our hybrid camera combines a Point Grey Dragonfly II camera, which captures images of  $1024 \times 768$  resolution at 25 fps (6.25 fps for image deblurring examples), and a Mikrotron MC1311 camera that captures images of  $128 \times 96$  resolution at 100 fps. A beamsplitter is employed to align their optical axes and respective images. Video synchronization is achieved using a 8051 microcontroller.

(HR-LFR) camera, whose images are blurred due to its slower temporal sampling. This ability to directly observe fast moving objects in the scene with the auxiliary camera allows us to handle a larger class of object motions without the use of prior motion models, since optical flow can be computed.

In the following, we discuss the construction of a hybrid camera, the optical flow and motion blur estimation, and the use of the low-resolution images as reconstruction constraints on highresolution images.

## A. Camera Construction

Three conceptual designs of the hybrid camera system were discussed by Ben-Ezra and Nayar [6]. In their work, they implemented a simple design in which the two cameras are placed side-by-side, such that their viewpoints can be considered the same when viewing a distant scene. A second design avoids the distant scene requirement by using a beam splitter to share between two sensing devices the light rays that pass through a single aperture, as demonstrated by McGuire *et al.* [36] for the studio matting problem. A promising third design



Fig. 4. Spatially-varying blur kernel estimation using optical flows. (a) Motion blur image. (b) Estimated blur kernels of (a) from optical flows.

is to capture both the HR-LFR and LR-HFR video on a single sensor chip. According to [9], this can readily be achieved using a programmable CMOS sensing device.

In our work, we constructed a hand-held hybrid camera system based on the second design as shown in Figure 3. The two cameras are positioned such that their optical axes and pixel arrays are well aligned. Video synchronization is achieved using a 8051 microcontroller. To match the color responses of the two devices we employ histogram equalization. In our implemented system, the exposure levels of the two devices are set to be equal, and the signal-to-noise ratios in the HR-LFR and LR-HFR images are approximately the same.

## B. Blur Kernel Approximation Using Optical Flows

In the absence of occlusion, disocclusion, and out-of-plane rotation, a blur kernel can be assumed to represent the motion of a camera relative to objects in the scene. In [6], this relative motion is assumed to be constant throughout an image, and the globally invariant blur kernel is obtained through the integration of global motion vectors over a spline curve.

However, since optical flow is in fact a local estimation of motions, we can calculate spatiallyvarying blur kernels from optical flows. We use the multiscale Lucas-Kanade algorithm [32] to calculate the optical flow at each pixel location. Following the brightness constancy assumption of optical flow estimation, we assume our motion blurred images are captured under constant illumination, such that the change of pixel color of moving scene/object points over the exposure period can be neglected. The per-pixel motion vectors are then integrated to form spatiallyvarying blur kernels, K(x, y), one per pixel. This integration is performed as described by [6] for global motion. We use a C1 continuity spline curve fit to the path of optical flow at position (x, y). The number of frame used to fit the spline curve is 16 for image examples and 4 for video examples (Figure 3). Figure 4 shows an example of spatially-varying blur kernels estimated from optical flows.

The optical flows estimated with the multiscale Lucas-Kanade algorithm [32] may contain noise that degrades blur kernel estimation. We found such noisy estimates to occur mainly in smooth or homogeneous regions which lack features for correspondence, while regions with sharp features tend to have accurate optical flows. Since deblurring artifacts are evident primarily around such features, the Lucas-Kanade optical flows are effective for our purposes. On the other hand, the optical flow noise in relatively featureless regions has little effect on deblurring results, since these areas are relatively unaffected by errors in the deblurring kernel. As a measure to heighten the accuracy and consistency of estimated optical flows, we use local smoothing [51] as an enhancement of the multiscale Lucas-Kanade algorithm [32].

The estimated blur kernels contain quantization errors due to the low resolution of the optical flows. Additionally, motion vector integration may provide an imprecise temporal interpolation of the flow observations. Our MAP optimization framework addresses these issues by refining the estimated blur kernels in addition to deblurring the video frames or images. Details of this kernel refinement will be discussed fully in Section IV.

## C. Back-Projection Constraints

The capture of low-resolution frames in addition to the high-resolution images not only facilitates optical flow computation, but also provides super-resolution-based reconstruction constraints [21], [22], [37], [10], [16], [3], [43] on the high-resolution deblurring solution. The back-projection algorithm [21], [22] is a common iterative technique for minimizing the reconstruction error, and can be formulated as follows:

$$I^{t+1} = I^t + \sum_{j=1}^{M} (u(W(I_{l_j}) - d(I^t \otimes h))) \otimes p$$
(1)





Fig. 5. Performance comparisons for different deconvolution algorithms on a synthetic example. The ground truth motion blur kernel is used to facilitate comparison. The signal-to-noise ratio (SNR) of each result is reported. (a) A motion blurred image [SNR(dB)=25.62] with the corresponding motion blur kernel shown in the inset. Deconvolution results using (b) Wiener filter [SNR(dB)=37.0], (c) Richardson-Lucy [SNR(dB)=33.89], (d) Total Variation Regularization [SNR(dB)=36.13], (e) Gradient Sparsity Prior [SNR(dB)=46.37], and (f) our approach [SNR(dB)=50.26dB], which combines constraints from both deconvolution and super-resolution. The low resolution image in (g) is 8x downsampled from the original image, shown in (h).

where M represents the number of corresponding low-resolution observations, t is an iteration index,  $I_{l_j}$  refers to the *j*-th low-resolution image,  $W(\cdot)$  denotes a warp function that aligns  $I_{l_j}$  to a reference image,  $\otimes$  is the convolution operation, h is the convolution filter before downsampling, p is a filter representing the back-projection process, and  $d(\cdot)$  and  $u(\cdot)$  are the downsampling and upsampling processes respectively. Equation (1) assumes that each observation carries the same weight. In the absence of a prior, h is chosen to be a Gaussian filter with a size proportionate to the downsampling factor, and p is set equal to h.

In the hybrid camera system, a number of low-resolution frames are captured in conjunction with each high-resolution image. To exploit this available data, we align these frames according to the computed optical flows, and use them as back-projection constraints in Equation (1).

The number of low-resolution image constraints M is determined by the relative frame rates of the cameras. In our implementation, we choose the first low-resolution frame as the reference frame to which the estimated blur kernel and other low-resolution frames are aligned. Choosing a different low-resolution frame as the reference frame would lead to different deblurred result, which is a property that can be used to increase the temporal samples of the deblurred video as later discussed in Section VI.

The benefits of using the back-projection constraint, and multiple such back-projection constraints, is illustrated in Figure 5. Each of the low-resolution frames presents a physical constraint on the high-resolution solution in a manner that resembles how each offset image is used in a super-resolution technique. The effectiveness of incorporating the back-projection constraint to suppress ringing artifacts is demonstrated in Figure 5 in compared to several other deconvolution algorithms.

#### IV. OPTIMIZATION FRAMEWORK

Before presenting our deblurring framework, we briefly review the Richardson-Lucy deconvolution algorithm, as our approach is fashioned in a similar manner. For the sake of clarity, our approach is first discussed for use in correcting global motion blur. This is followed by its extension to spatially-varying blur kernels.

## A. Richardson-Lucy Image Deconvolution

The Richardson-Lucy algorithm [40], [33] is an iterative maximum likelihood deconvolution algorithm derived from Bayes' theorem that minimizes the following estimation error:

$$\arg\min_{I} n(||I_b - I \otimes K||^2) \tag{2}$$

where I is the deblurred image, K is the blur kernel,  $I_b$  is the observed blur image, and  $n(\cdot)$  is the image noise distribution. A solution can be obtained using the iterative update algorithm defined as follows:

$$I^{t+1} = I^t \times K * \frac{I_b}{I^t \otimes K}$$
(3)

where \* is the correlation operation. A blind deconvolution method using the Richardson-Lucy algorithm was proposed by Fish *et al.* [18], which iteratively optimizes *I* and *K* in alternation using Equation (3) with the positions of *I* and *K* switched during optimization iterations for



Fig. 6. Multiscale refinement of a motion blur kernel for the image in Figure 11. (a) to (e) exhibit refined kernels at progressively finer scales. Our kernel refinement starts from the coarsest level. The result of each coarser level is then upsampled and used as an initial kernel estimate for the next level of refinement.

K. The Richardson-Lucy algorithm assumes image noise  $n(\cdot)$  to follow a Poisson distribution. If we assume image noise to follow a Gaussian distribution, then a least squares method can be employed [21]:

$$I^{t+1} = I^t + K * (I_b - I^t \otimes K) \tag{4}$$

which shares the same iterative back-projection update rule as Equation (1).

From video input with computed optical flows, multiple blurred images  $I_b$  and blur kernels K may be acquired by reversing the optical flows of neighboring high-resolution frames. These multiple observation constraints can be jointly applied in Equation (4) [39] as

$$I^{t+1} = I^t + \sum_{i=1}^{N} w_i K_i * (I_{b_i} - I^t \otimes K_i)$$
(5)

where N is the number of aligned observations. That image restoration can be improved with additional observations under different motion blurs is a important property that we exploit in this work. The use of neighboring frames in this manner may also serve to enhance the temporal consistency of the deblurred video frames.

# B. Optimization for Global Kernels

In solving for the deblurred images, our method jointly employs the multiple deconvolution and back-projection constraints available from the hybrid camera input. For simplicity, we assume in this subsection that the blur kernels are spatially invariant. Our approach can be formulated



Fig. 7. Convolution with kernel decomposition. (a) Convolution result without kernel decomposition, where full blur kernels are generated on-the-fly per pixel using optical flow integration. (b) Convolution using 30 PCA-decomposed kernels. (c) Convolution using a patch-based decomposition. (d) Convolution using delta function decomposition of kernels, with at most 30 delta functions per pixel.



Fig. 8. PCA versus the delta function representation for kernel decomposition. The top row illustrates the kernel decomposition using PCA, and the bottom row shows the decomposition using the delta function representation. The example kernel is taken from among the spatially-varying kernels of Figure 7, from which the basis kernels are derived. Weights are displayed below each of the basis kernels. The delta function representation not only guarantees positive values of basis kernels, but also provides more flexibility in kernel refinement.

into a MAP estimation framework as follows:

$$\arg \max_{I,K} P(I, K|I_b, K_o, I_l)$$

$$= \arg \max_{I,K} P(I_b|I, K) P(K_o|I, K) P(I_l|I) P(I) P(K)$$

$$= \arg \min_{I,K} L(I_b|I, K) + L(K_o|I, K) + L(I_l|I) + L(I) + L(K)$$
(6)

where I and K denote the sharp images and the blur kernels we want to estimate,  $I_b$ ,  $K_o$ and  $I_l$  are the observed blur images, estimated blur kernels from optical flows, and the lowresolution, high-frame-rate images respectively, and  $L(\cdot) = -log(P(\cdot))$ . In our formulation, the priors P(I) and P(K) are taken to be uniformly distributed. Assuming that  $P(K_o|I, K)$  is conditionally independent of I, that the estimation errors of likelihood probabilities  $P(I_b|I, K)$ ,  $P(K_o|I, K)$  and  $P(I_l|I)$  follow Gaussian distributions and that each observation of  $I_b$ ,  $K_o$  and  $I_l$  is independent and identically distributed, we can then rewrite Equation (6) as

$$\arg\min_{I,K} \sum_{i}^{N} ||I_{b_{i}} - I \otimes K_{i}||^{2} + \lambda_{B} \sum_{j}^{M} ||I_{l_{j}} - d(I \otimes h)||^{2} + \lambda_{K} \sum_{i}^{N} ||K_{i} - K_{o_{i}}||^{2}$$
(7)

where  $\lambda_K$  and  $\lambda_B$  are the relative weights of the error terms. To optimize the above equation for *I* and *K*, we employ alternating minimization. Combining Equations (1) and (5) yields our iterative update rules:

1) Update  $I^{t+1} = I^t + \sum_{i=1}^N K_i^t * (I_{b_i} - I^t \otimes K_i^t) + \lambda_B \sum_{j=1}^M h \otimes (u(W(I_{l_j}) - d(I^t \otimes h)))$ 2) Update  $K_i^{t+1} = K_i^t + \widetilde{I}^{t+1} * (I_{b_i} - I^{t+1} \otimes K_i^t) + \lambda_K (K_{o_i} - K_i^t)$ 

where  $\tilde{I} = I / \sum_{(x,y)} I(x,y)$ ,  $I(x,y) \ge 0$ ,  $K_i(u,v) \ge 0$ , and  $\sum_{(u,v)} K_i(u,v) = 1$ . The two update steps are processed in alternation until the change in I falls below a specified level or until a maximum number of iterations is reached. The term  $W(I_{l_j})$  is the warped aligned observations. The reference frame to which these are aligned to can be any of the M low-resolution images. Thus for each deblurred high-resolution frame we have up to M possible solutions. This will later be used in the temporal upsampling described in Section VI. In our implementation, we set N = 3 in correspondence to the current, previous and next frames, and M is set according to the relative camera settings (4/16 for video/image deblurring in our implementation). We also initialize  $I^0$  as the currently observed blur image  $I_b$ ,  $K_i^0$  as the estimated blur kernel  $K_{o_i}$  from optical flows, and set  $\lambda_B = \lambda_K = 0.5$ .

For more stable and flexible kernel refinement, we refine the kernel in a multiscale fashion as done in [17], [52]. Figure 6 illustrates the kernel refinement process. We estimate PSFs from optical flows of the observed low-resolution images and then downsample to the coarsest level. After refinement at a coarser level, kernels are then upsampled and refined again. The multiscale pyramid is constructed using a downsampling factor of  $1/\sqrt{2}$  with five levels. The likelihood  $P(K_o|K)$  is applied at each level of the pyramid with a decreasing weight, so as to allow more flexibility in refinement at finer levels. We note that starting at a level coarser than the lowresolution images allows our method to recover from some error in PSF estimation from optical flows.

# C. Spatially-varying Kernels

A spatially-varying blur kernel can be expressed as K(x, y, u, v), where (x, y) is the image coordinate and (u, v) is the kernel coordinate. For large sized kernels, e.g.,  $65 \times 65$ , this representation is impractical due to its enormous storage requirements. Recent work has suggested ways to reduce the storage size, such as by constraining the motion path [42]; however, our approach places no constraints on possible motion. Instead, we decompose the spatially-varying kernels into a set of P basis kernels  $k_l$  whose mixture weights  $a_l$  are a function of image location:

$$K(x, y, u, v) = \sum_{l=1}^{P} a_l(x, y) k_l(u, v).$$
(8)

The convolution equation then becomes

$$I(x,y) \otimes K(x,y,u,v) = \sum_{l=1}^{P} a_l(x,y)(I(x,y) \otimes k_l(u,v)).$$
(9)

In related work [26], principal components analysis (PCA) is used to determine the basis kernels. PCA, however, does not guarantee positive kernel values, and we have found in our experiments that PCA-decomposed kernels often lead to unacceptable ringing artifacts, exemplified in Figure 7(b). The ringing artifacts in the convolution result resemble the patterns of basis kernels. Another method is to use a patch representation which segments images into many small patches such that the local motion blur kernel is the same within each small patch. This method was used in [25], but their blur kernels are defocus kernels with very small variations within local areas. For large object motion, blur kernels in the patch based method would not be accurate, leading to discontinuity artifacts as shown in Figure 7(c). We instead choose to use a delta function representation, where each delta function represents a position (u, v) within a kernel. Since a motion blur kernel is typically sparse, we store only  $30 \sim 40$  delta functions for each image pixel, where the delta function positions are determined by the initial optical flows. From the total  $65 \times 65$  possible delta-function in the spatial kernel at each pixel in the image, we find in practice that we only use about  $500 \sim 600$  distinct delta functions to provide a sufficient approximation of the spatially-varying blur kernels in the convolution process. Examples of basis

kernel decomposition using PCA and the delta function representation are shown in Figure 8. The delta function representation also offers more flexibility in kernel refinement, while refinements using the PCA representation are limited to the PCA subspace.

Combining Equations (9) and (7), our optimization function becomes

$$\arg\min_{I,K} \qquad \sum_{i}^{N} ||I_{b_{i}} - \sum_{l}^{P} a_{il}(I \otimes k_{il})||^{2} + \lambda_{B} \sum_{j}^{M} ||I_{l_{j}} - d(I \otimes h)||^{2} + \lambda_{K} \sum_{i}^{N} \sum_{l}^{P} ||a_{il}k_{il} - a_{o_{il}}k_{il}||^{2}.$$
(10)

The corresponding iterative update rules are then

- 1) Update  $I^{t+1} = I^t + \sum_{i=1}^N \sum_l^P a^t_{il} k_{il} * (I_{b_i} \sum_l^P a^t_{il} (I^t \otimes k_{il})) + \lambda_B \sum_{j=1}^M h \otimes (u(W(I_{l_j}) d(I^t \otimes h)))$
- 2) Update  $a_{il}^{t+1} = a_{il}^t + (\widetilde{I'}^{t+1} * (I'_{b_i} \sum_l^P a_{il}^t (I'^{t+1} \otimes k_{il}))) \cdot k_{il} + \lambda_K (a_{o_{il}} a_{il}^t)$

where I' and  $I'_b$  are local windows in the estimated result and the blur image. This kernel refinement can be implemented in a multiscale framework for greater flexibility and stability. The number of delta functions  $k_{il}$  stored at each pixel position may be reduced when an updated value of  $a_{il}$  becomes insignificant. For greater stability, we process each update rule five times before switching to the other.

## D. Discussion

Utilizing both deconvolution of high-resolution images and back-projection from low-resolution images offers distinct advantages, because the deblurring information from these two sources tend to complement each other. This can be intuitively seen by considering a low-resolution image to be a sharp high-resolution image that has undergone motion blurring with a Gaussian PSF and bandlimiting. Back-projection may then be viewed as a deconvolution with a Gaussian blur kernel that promotes recovery of lower-frequency image features without artifacts. On the other hand, deconvolution of high-resolution images with the high-frequency PSFs typically associated with camera and object motion generally supports reconstruction of higher-frequency details, especially those orthogonal to the motion direction. While some low-frequency content can also be restored from motion blur deconvolution, there is often significant loss due to the large support regions for motion blur kernels, and this results in ringing artifacts. As discussed in [39], the joint use of images having such different blur functions and deconvolution information favors a better deblurring solution.

Multiple motion blur deconvolutions and multiple back-projections can further help to generate high quality results. Differences in motion blur kernels among neighboring frames provide different frequency information; and multiple back-projection constraints help to reduce quantization and the effects of noise in low-resolution images. In some circumstances there exists redundancy in information from a given source, such as when high-resolution images contain identical motion blur, or when low-resolution images are offset by integer pixel amounts. This makes it particularly important to utilize as much deblurring information as can be obtained.

Our current approach does not utilize priors on the deblurred image or the kernels. With constraints from the low-resolution images, we have found these priors to be unneeded. Figure 5 compares our approach with other deconvolution algorithms. Specifically, we compare our approach with Total Variation regularization [15] and Sparsity Priors [28], which have recently been shown to produce better results than traditional Wiener filtering [50] and the Richardson-Lucy [40], [33] algorithm. Both Total Variation regularization and Sparsity Priors produce results with less ringing artifacts. There are almost no ringing artifacts with Sparsity Priors, but many fine details are lost. In our approach, most medium to large scale ringing artifacts are removed using the back-projection constraints, while fine details are recovered through deconvolution.

Although our approach can acquire and utilize a greater amount of data, high-frequency details that have been lost by both motion blur and downsampling cannot be recovered. This is a fundamental limitation of any deconvolution algorithm that does not hallucinate detail. We also note that reliability in optical flow cannot be assumed beyond a small time interval. This places a restriction on the number of motion blur deconvolution constraints that can be employed to deblur a given frame.

Lastly, we note that iterative back-projection technique incorporated into our framework is known to have convergence problems. Empirically we have found that stopping after no more than 50 iterations of our algorithm produces acceptable results.

## V. DEBLURRING OF MOVING OBJECTS

To deblur a moving object, a high-resolution image needs to be segmented into different layers because pixels on the blended boundaries of moving objects contain both foreground and background components, each with different relative motion to the camera. This layer separation



Fig. 9. Layer separation using a hybrid camera: (a)-(d) Low-resolution frames and their corresponding binary segmentation masks. (e) High-resolution frame and the matte estimated by compositing the low-resolution segmentation masks with smoothing.

is inherently a matting problem that can be expressed as:

$$I = \alpha F + (1 - \alpha)B \tag{11}$$

where I is the observed image intensity, F, B and  $\alpha$  are the foreground color, background color and alpha value of the fractional occupancy of the foreground. The matting problem is an ill-posed problem since the number of unknown variables is greater than the number of observations. Single image approaches require user assistance to provide a trimap [14], [13], [46] or scribbles [49], [29], [48] for collecting samples of the foreground and background colors. Fully automatic approaches, however, have required either a blue background [44], multiple cameras with different focus [35], polarized illumination [36] or a camera array [24]. In this section, we propose a simple solution to the layer separation problem that takes advantage of the hybrid camera system.

Our approach assumes that object motion does not cause motion blur in the high-frame-rate camera, such that the object appears sharp. To extract the alpha matte of a moving object, we perform binary segmentation of the moving object in the low-resolution images, and then compose the binary segmentation masks with smoothing to approximate the alpha matte in the

high-resolution image. We note that Ben-Ezra and Nayar [7] used a similar strategy to perform layer segmentation in their hybrid-camera system. In Figure 9, an example of this matte extraction is demonstrated together with the moving object separation method of Zhang *et al.* [54]. The foreground color F also must be estimated for deblurring. This can be done by assuming a local color smoothness prior on F and B and solving for their values with Bayesian matting [14]:

$$\begin{bmatrix} \Sigma_F^{-1} + \mathbf{I}\alpha^2/\sigma_I^2 & \mathbf{I}\alpha(1-\alpha)/\sigma_I^2 \\ \mathbf{I}\alpha(1-\alpha)/\sigma_I^2 & \Sigma_B^{-1} + \mathbf{I}(1-\alpha)^2/\sigma_I^2 \end{bmatrix} \begin{bmatrix} F \\ B \end{bmatrix} = \begin{bmatrix} \Sigma_F^{-1}\mu_F + I\alpha/\sigma_I^2 \\ \Sigma_B^{-1}\mu_B + I(1-\alpha)/\sigma_I^2 \end{bmatrix}$$
(12)

where  $(\mu_F, \Sigma_F)$  and  $(\mu_B, \Sigma_B)$  are the local color mean and covariance matrix (Gaussian distribution) of the foreground and background colors, I is a 3 × 3 identity matrix, and  $\sigma_I$  is the standard derivation of I, which models estimation errors of Equation (11). Given the solution of F and B, the  $\alpha$  solution can be refined by solving Equation (11) in closed form. Refinements of F, B, and  $\alpha$  can be done in alternation to further improve the result.

Once moving objects are separated, we deblur each layer separately using our framework. The alpha mattes are also deblurred for compositing, and the occluded background areas revealed after alpha mask deblurring can then be filled in either by back-projection from the low-resolution images or by the motion inpainting method of [34].

## VI. TEMPORAL UPSAMPLING

Unlike deblurring of images, videos require deblurring of multiple consecutive frames in a manner that preserves temporal consistency. As described in Section IV-B, we can jointly use the current, previous and subsequent frames to deblur the current frame in a temporally consistent way. However, after sharpening each individual frame, temporal discontinuities in the deblurred high-resolution, low-frame-rate video may become evident through some jumpiness in the sequence. In this section, we describe how our method can alleviate this problem by increasing the temporal sampling rate to produce a deblurred high-resolution, high-frame-rate video.

As discussed by Shechtman *et al.* [43], temporal super resolution results when an algorithm can generate an output with a temporal rate that surpasses the temporal sampling of any of the input devices. While our approach generates a high-resolution video at greater temporal rate than the input high-resolution, low-frame-rate video, its temporal rate is bounded by the frame rate



Fig. 10. Relationship of high-resolution deblurred result to corresponding low-resolution frame. Any of the low-resolution frame can be selected as a reference frame for the deblurred result. This allows up to M deblurred solutions to be obtained.

the low-resolution, high-frame-rate camera. We therefore refrain from the term super resolution and refer to this as *temporal upsampling*.

Our solution to temporal upsampling derives directly from our deblurring algorithm. The deblurring problem is a well-known under-constrained problem since there exist many solutions that can correspond to a given motion blurred image. In our scenario, we have M high-frame-rate low-resolution frames corresponding to each high-resolution, low-frame-rate motion blurred image. Figure 10 shows an example. With our algorithm, we therefore have the opportunity to estimate M solutions using each one of the M low-resolution frames as the basic reference frame. While the ability to produce multiple deblurred frames is not a complete solution to temporal upsampling, here the use of these M different reference frames leads to a set of deblurred frames that is consistent with the temporal sequence. This unique feature of our approach is gained through the use of the hybrid camera to capture low-resolution, high-frame-rate video in addition to the standard high-resolution, low-frame-rate video. The low-resolution, high-frame-rate video in as the basic network of the deblurred video frame rate. The result is a high-resolution, high-frame-rate deblurred video.





Fig. 11. Image deblurring using globally invariant kernels. (a) Input. (b) Result generated with the method of [17], where the user-selected region is indicated by a black box. (c) Result generated by [6]. (d) Result generated by back-projection [21]. (e) Our results. (f) The ground truth sharp image. Close-up views and the estimated global blur kernels are also shown.

## VII. RESULTS AND COMPARISONS

We evaluate our deblurring framework using real images and videos. In these experiments, a ground-truth blur-free image is acquired by mounting the camera on a tripod and capturing a static scene. Motion blurred images are then obtained by moving the camera and/or introducing a dynamic scene object. We show examples of several different cases: globally invariant motion blur caused by hand shake, in-plane rotational motion of a scene object, translational motion of a scene object, out-of-plane rotational motion of an object, zoom-in motion caused by changing the focal length (i.e. camera's zoom setting), a combination of translational motion and rotational motion with multiple frames used as input for deblurring one frame, video



Fig. 12. Image deblurring with spatial varying kernels from rotational motion. (a) Input. (b) Result generated with the method of [42] (obtained courtesy of the authors of [42]). (c) Result generated by [6] using spatially-varying blur kernels estimated from optical flow. (d) Result generated by back-projection [21]. (e) Our results. (f) The ground truth sharp image. Close-ups are also shown.

# deblurring with out-of-plane rotational motion, video deblurring with complex in-plane motion, and video deblurring with a combination of translational and zoom-in motion.

[Globally invariant motion blur] In Figure 11, we present an image deblurring example with globally invariant motion, where the input is one high-resolution image and several low-resolution images. Our results are compared with those generated by the methods of Fergus *et al.* [17], Ben-Ezra and Nayar [6] and back projection [21]. Fergus *et al.*'s approach is a state-of-the-art blind deconvolution technique that employs a natural image statistics constraint. However, when the blur kernel is not correctly estimated, an unsatisfactory result shown in (b)





Fig. 13. Image deblurring with translational motion. In this example, the moving object is a car moving horizontally. We assume that the motion blur within the car is globally invariant. (a) Input. (b) Result generated by [17], where the user-selected region is indicated by the black box. (c) Result generated by [6]. (d) Result generated by back-projection [21]. (e) Our results. (f) The ground truth sharp image captured from another car of the same model. Close-up views and the estimated global blur kernels within the motion blur layer are also shown.

will be produced. Ben-Ezra and Nayar use the estimated optical flow as the blur kernel and then perform deconvolution. Their result in (c) is better than that in (b) as the estimated blur kernel is more accurate, but ringing artifacts are still unavoidable. Back-projection produces a superresolution result from a sequence of low resolution images as shown in (d). Noting that motion blur removal is not the intended application of back-projection, we can see that its results are blurry since the high-frequency details are not sufficiently captured in the low-resolution images. The result of our method and the refined kernel estimate are displayed in (e). The ground truth



Fig. 14. Image deblurring with spatially-varying kernels. In this example, the moving object contains out-of-plane rotation with both occlusion and disocclusion at the object boundary. (a) Input. (b) Result generated by [6]. (c) Result generated by back projection [21]. (d) Our results using the first low-resolution frame as the reference frame. (e) Our results using the last low-resolution frame as the reference frame. (f) The ground truth sharp image. Close-ups are also shown.

is given in (f) for comparison.

**[In-plane rotational motion]** Figure 12 shows an example with in-plane rotational motion. We compared our result with those by Shan *et al.* [42], Ben-Ezra and Nayar [6], and back-projection [21]. Shan *et al.* [42] is a recent technique that targets deblurring of in-plane rotational motion. Our approach is seen to produce less ringing artifacts compared to [42] and [6], and it generates greater detail than [21].

[Translational motion] Figure 13 shows an example of a car translating horizontally. We assume the motion blur within the car region is globally invariant and thus techniques for





Fig. 15. Image deblurring with spatially-varying kernels. In this example, the camera is zooming into the scene. (a) Input. (b) Result generated by [17]. (b) Result generated by [6]. (c) Result generated by back-projection [21]. (d) Our results. (f) The ground truth sharp image. Close-ups are also shown.

removing globally invariant motion blur can be applied after layer separation of the moving object. We use the technique proposed in Section V to separate the moving car from the static background. Our results are compared with those generated by Fergus *et al.* [17], Ben-Ezra and Nayar [6] and back-projection [21]. In this example, the moving car is severely blurred with most of the high frequency details lost. We demonstrate in (c) the limitation of using deconvolution alone even with an accurate motion blur kernel. In this example, the super-resolution result in (d) is better than the deconvolution result, but there are some high-frequency details that are not recovered. Our result is shown in (e) which maintains most low-frequency details recovered by super-resolution and also high-frequency details recovered by deconvolution. Some incorrect





Fig. 16. Deblurring with and without multiple high-resolution frames. (a)(b) Input images containing both translational and rotational motion blur. (c) Deblurring using only (a) as input. (d) Deblurring using only (b) as input. (e) Deblurring of (a) using both (a) and (b) as inputs. (f) Ground truth sharp image. Close-ups are also shown.

high-frequency details from the static background are incorrectly retained in our final result, because of the presence of some high-frequency background details in the separated moving object layer. We believe that a better layer separation algorithm would lead to improved results. This example also exhibits a basic limitation of our approach. Since there is significant car motion during the exposure time, most high frequency detail is lost and cannot be recovered by our approach. The ground truth in (f) shows a similar, parked car for comparison.

**[Out-of-plane rotational motion]** Figure 14 shows an example of out-of-plane rotation where occlusion/disocclusion occurs at the object boundary. Our result is compared to that of Ben-Ezra and Nayar [6] and back-projection [21]. One major advantage of our approach is that we can

detect the existence of occlusions/disocclusions of the motion blurred moving object. This not only helps to estimate the alpha mask for layer separation, but also aids in eliminating irrelevant low-resolution reference frame constraints for back projection. We show our result by choosing the first frame and the last frame as the reference frame. Both occlusion and disocclusion are contained in this example.

[Zoom-in motion] Figure 15 shows another example of motion blur from zoom-in effects. Our result is compared to Fergus *et al.* [17], Ben-Ezra and Nayar [6] and back-projection [21]. We note that the method of Fergus *et al.* [17] is intended for globally invariant motion blur, and is shown here to demonstrate the effects of using only a single blur kernel to deblur spatiallyvarying motion blur. Again, our approach produces better results with less ringing artifacts and richer detail.

[**Deblurring with multiple frames**] The benefit of using multiple deconvolutions from multiple high-resolution frames is exhibited in Figure 16 for a pinwheel with both translational and rotational motion. The deblurring result in (c) was computed using only (a) as input. Likewise, (d) is the deblurred result from only (b). Using both (a) and (b) as inputs yields the improved result in (e). This improvement can be attributed to the difference in high-frequency detail that can be recovered from each of the differently blurred images. The ground truth is shown in (f) for comparison.

[Video deblurring with out-of-plane rotational motion] Figure 17 demonstrates video deblurring of a vase with out-of-plane rotation. The center of rotation is approximately aligned with the image center. The top row displays five consecutive input frames. The second row shows close-ups of a motion blurred region. The middle row shows our results with the first low-resolution frames as the reference frames. The fourth and fifth rows show close-ups of our results with respect to the first and the fifth low-resolution frames as the reference frames.

This example also demonstrates the ability to produce multiple deblurring solutions as described in Section VI. For temporal super-resolution, we combine the results together in the order indicated by the red lines in Figure 17. With our method, we can increase the frame rate of deblurred high resolution videos up to the same rate as the low-resolution, high-frame-rate video input.

[Video deblurring with complex in-plane motion] Figure 18 presents another video deblurring result of a tossed box with complex (in-plane) motion. The top row displays five consecutive



Fig. 17. Video deblurring with out-of-plane rotational motion. The moving object is a vase with a center of rotation approximately aligned with the image center. First Row: Input video frames. Second Row: Close-ups of a motion blurred region. Third Row: Deblurred video. Fourth Row: Close-ups of deblurred video using the first low-resolution frames as the reference frames. Fifth Row: Close-ups of deblurred video frames using the fifth low-resolution frames as the reference frames. The final video sequence has higher temporal sampling than the original high-resolution video, and is played with frames ordered according to the red lines.

input frames. The second row shows close-ups of the motion blurred moving object. The middle row shows our separated mattes for the moving object, and the fourth and the fifth rows present our results with the first and third low-resolution frames as reference. The text on the tossed box is recovered to a certain degree by our video deblurring algorithm. Similar to the previous video deblurring example, our output is a high-resolution, high-frame-rate deblurred video. This result also illustrates a limitation of our method, where the shadow of the moving object is not deblurred and may appear inconsistent. This problem is a direction for future investigation.

[Video deblurring with a combination of translational and zoom-in motion] Our final example is shown in Figure 19. The moving object of interest is a car driving towards the camera.



Fig. 18. Video deblurring with a static background and a moving object. The moving object is a tossed box with arbitrary (in-plane) motion. First Row: Input video frames. Second Row: Close-up of the motion blurred moving object. Third Row: Extracted alpha mattes of the moving object. Fourth Row: The deblurred video frames using the first low-resolution frames as the reference frames. Fifth Row: The deblurred video frames using the third low-resolution frames as the reference frames. The final video with temporal super-resolution is played with frames ordered as indicated by the red lines.

Both translational effects and zoom-in blur effects exist in this video deblurring example. The top row displays five consecutive frames of input. The second row shows close-ups of the motion blurred moving object. The middle row shows our extracted matters for the moving object, and the fourth and the fifth rows present our results with the first and the fifth low-resolution frames as reference.

# VIII. CONCLUSION

We have proposed an approach for image/video deblurring using a hybrid camera. Our work has formulated the deblurring process as an iterative method that incorporates optical flow, backprojection, kernel refinement, and frame coherence to effectively combine the benefits of both



Fig. 19. Video deblurring in an outdoor scene. The moving object is a car driving towards the camera, which produces both translation and zoom-in blur effects. First Row: Input video frames. Second Row: The extracted alpha mattes of the moving object. Third Row: The deblurred video frames using the first low-resolution frames as the reference frames. Fourth Row: The deblurred video frames using the third low-resolution frames as the reference frames. The final video consists of frames ordered as indicated by the red lines. By combining results from using different low-resolution frames as reference frames, we can increase the frame rate of the deblurred video.

deconvolution and super-resolution. We demonstrate that this approach can produce results that are sharper and cleaner than state-of-the-art techniques.

While our video deblurring algorithm exhibits high-quality results on various scenes, there exist complicated forms of spatially-varying motion blur that can be difficult for our method to handle (e.g., motion blur effects caused by object deformation). The performance of our algorithm is also bounded by the performance of several of its components, including optical flow estimation, layer separation and also the deconvolution algorithm. Despite these limitations, we have proposed the first work to handle spatially-varying motion blur with arbitrary in-plane/out-of-plane rigid motion. This work is also the first to address video deblurring and to increase video frame rates

using a deblurring algorithm.

Future research directions for this work include how to improve the deblurring performance through incorporating priors into our framework. Recent deblurring methods have demonstrated the utility of priors, such as the natural image statistics prior and the sparsity prior, for reducing ringing artifacts and for kernel estimation. Another research direction is to improve layer separation by more fully exploiting the available information in the hybrid camera system. Additional future work may also be done on how to recover the background partially occluded by a motion blurred object.

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